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Vibration Characteristics of Partially Covered Double-Sandwich Cantilever Beam

Qinghua Chen* and Cesar Levy†
Florida International University, Miami, Florida 33199

The differential equations of motion together with the boundary conditions for a partially covered, double-sandwich cantilever beam are derived. Bending and extension, rotational and longitudinal inertia of damping layers, and shear deformation and rotational and longitudinal inertia of the constraining layers and the primary beam are included in the equations. The theory is applicable for long as well as short, soft, or stiff damping layer, double-sandwich beams. Also, the effects of different parameters on the system loss factor and resonance frequency are discussed. Differences are found to exist with the previous beam model (called the Euler beam model) when the damping layers are stiff, when the thickness of the damping layer is large compared to the primary-beam thickness, and in the case of higher modes of vibration.

Introduction

BECAUSE of their high energy dissipation ability, constrained or unconstrained viscoelastic damping treatments have been used for noise and vibration control for many years. Many improvements have been suggested about the application of such structures and many papers have been published since Kerwin's publication on the vibration of three-layered beams.[1] Rao,[2] Ditaranto,[3] and Mead[4] derived a sixth-order differential equation of motion for a sandwich beam via different methods. The effects of different boundary conditions were studied in these papers. The vibration of a short sandwich beam also was discussed by Rao[5] but only in the simply supported beam case. The random vibration of a sandwich beam subjected to base acceleration excitation was studied by Xi et al.[6] Chen and Xi[7] obtained an improved equation for the sandwich beam.

Because of the consideration of structure weight and cost of materials involved in fully covering the beam with viscoelastic material, partially covered sandwich beams and plates were investigated by Lall et al.[8] and Garrison et al.[9] To increase the damping, multiple sandwich structures were presented by many authors.[10–13] Also, vibration properties of a partially covered, double-sandwich cantilever beam were studied by Levy and Chen.[14] The bonded joints with viscoelastic damping treatment were studied by Zhou and Rao[15] and He and Rao.[16] The vibration characteristics of a double-sandwich cantilever beam with concentrated mass at the free end were studied by Chen and Levy.[17] These papers also discussed the effects of different parameters on system loss factor and resonance frequency. However, many effects, such as the shear deformation in constraining layers and the primary beam and the rotational and longitudinal inertias of all layers, were not included in the papers.

In this investigation, the nondimensional form of improved differential equations of motion for a partially covered, double-sandwich cantilever beam are derived and the equations are expressed in matrix form. It is not practical to obtain the differential equation in terms of a single variable. However, inspection shows that the expansion in a single parameter would lead to an equation that is of 14th order in both $\partial/\partial x$ and $\partial/\partial t$. The effects of different parameters on resonance frequency and loss factor are investigated.

Theoretical Model

The geometry of the double-sandwich cantilever beam is shown in Figs. 1 and 2. The following assumptions are made in the analysis of the damped part: 1) the beam deflection is uniform across any section; 2) no slip occurs at the interface between layers; and 3) plane cross sections in individual layers continue to remain plane after deformation.

Let u_i ($i = 1, 2, \ldots, 5$) be the longitudinal displacement of the ith neutral axis, γ_i ($i = 1, 2, \ldots, 5$) be the shear at the neutral axis, and ψ_i ($i = 1, 2, \ldots, 5$) denote the slope of the deflection curve due to bending. We have the following relationships:

$$\left.\begin{aligned} u_{x_i} &= u_i + z_i \psi_i \\ u_{z_i} &= w(x, t) \end{aligned}\right\} \qquad (i = 1, 2, \ldots, 5) \tag{1}$$

Where u_{x_i} and u_{z_i} represent the longitudinal and transverse displacement in each layer, respectively. The second expression of Eq. (1) is an outcome of the first assumption. From assumption (2), we have

$$u_i + h_i \psi_i = u_{i+1} - h_{i+1} \psi_{i+1}, \qquad i = 1, \ldots, 4 \tag{2a}$$

where h_i ($i = 1, 2, \ldots, 5$) is the half height of the ith layer. By defining k' as the Timoshenko constant; L_1 and L as the lengths of damping coverage and the whole beam, respectively; m_i and A_i as the mass per unit length and the cross-sectional area of the ith layer, respectively; E_i and G_i as the Young's modulus and the shear modulus of the ith layer; and by letting

$$\bar{x} = x/L \qquad \bar{w} = w/L \qquad \bar{u}_i = u_i/L \qquad H_i = h_i/L \tag{2b}$$

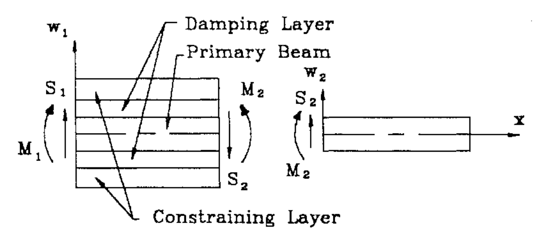

Fig. 1 Double-sandwich cantilever beam.

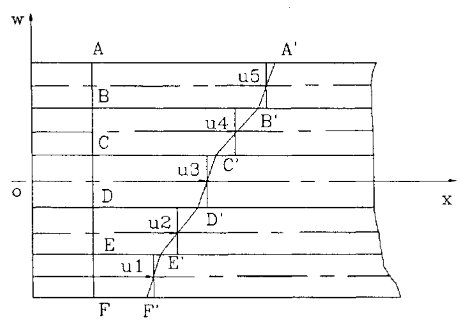

Fig. 2 Geometry of the beam.

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*Research Associate, Mechanical Engineering Department.
†Professor, Mechanical Engineering Department.

Eq. (2a) can be written as

$$\bar{u}_i + H_i\psi_i = \bar{u}_{i+1} - H_{i+1}\psi_{i+1}, \qquad i = 1,\dots,4 \tag{2c}$$

By defining the following dimensionless parameters for layer $i = 1,\dots,5$:

$$\bar{t} = t/t_0 \qquad t_0 = \sqrt{mL^2/EA}$$

$$m = \sum_{i=1}^{5} m_i \qquad EA = \sum_{i=1}^{5} E_i A_i$$

$$M_i = m_i/m \qquad \bar{E}_i = E_i A_i/EA$$

$$\bar{G}_i = \frac{k_i' G_i A_i}{EA} \qquad l = \frac{L_1}{L}$$

the kinetic and potential energy of all layers together yields

$$T = \frac{mL^3}{6t_0^2}\sum_{i=1}^{5}\int_0^l M_i\left[3\left(\frac{\partial \bar{u}_i}{\partial \bar{t}}\right)^2 + H_i^2\left(\frac{\partial \psi_i}{\partial \bar{t}}\right)^2 + 3\left(\frac{\partial \bar{w}}{\partial \bar{t}}\right)^2\right]\mathrm{d}\bar{x} \tag{3}$$

$$P = \frac{EAL}{6}\sum_{i=1}^{5}\int_0^l \left\{\bar{E}_i\left[3\left(\frac{\partial \bar{u}_i}{\partial \bar{x}}\right)^2 + H_i^2\left(\frac{\partial \psi_i}{\partial \bar{x}}\right)^2\right] + 3\bar{G}_i\left(\frac{\partial \bar{w}}{\partial \bar{x}} - \psi_i\right)^2\right\}\mathrm{d}\bar{x} \tag{4}$$

By Hamilton's principle, we have

$$\delta\int_{t_1}^{t_2}(T - P)\,\mathrm{d}t = 0 \tag{5}$$

Substituting Eqs. (3) and (4) into Eq. (5) and employing the continuity relationships (2c) to express ψ_1, ψ_2, ψ_4, and ψ_5, in terms of $\bar{u}_i$ and ψ_3 yields the following form of the differential equations of motion for section 1 (covered part):

$$[M]\{\ddot{Y}\} + [K]\{Y\} = 0 \tag{6}$$

where

$$\{Y\} = \{\bar{u}_1, \bar{u}_2, \bar{u}_3, \bar{u}_4, \bar{u}_5, \psi_3, \bar{w}\}^T$$

$$[M] = \begin{Bmatrix} 6M_1 & -6M_1 & 3M_1 & 0 & 0 & -3M_1H_3 & 0 \\ & 12M_1 + 6M_2 & -(6M_1 + 3M_3) & 0 & 0 & (6M_1 + 3M_2)H_3 & 0 \\ & & 3 & -(3M_4 + 6M_5) & 3M_5 & -3H_2(M_1 + M_2 - M_4 - M_5) & 0 \\ & & & 6M_4 + 12M_5 & -6M_5 & -(3M_4 + 6M_5)H_3 & 0 \\ & \text{symmetric} & & & 6M_5 & 3M_5H_3 & 0 \\ & & & & & 3H_3^2 & 0 \\ & & & & & & 3 \end{Bmatrix}$$

and the elements of the K matrix are

$$k_{11} = 4\bar{E}_1\frac{\partial^2}{\partial \bar{x}^2} - \frac{3\bar{G}_1}{H_1^2} \qquad k_{12} = -2\bar{E}_1\frac{\partial^2}{\partial \bar{x}^2} + \frac{6\bar{G}_1}{H_1^2}$$

$$k_{13} = \bar{E}_1\frac{\partial^2}{\partial \bar{x}^2} - \frac{3\bar{G}_1}{H_1^2} \qquad k_{14} = 0 \qquad k_{15} = 0$$

$$k_{16} = -\bar{E}_1H_3\frac{\partial^2}{\partial \bar{x}^2} + \frac{-3\bar{G}_1H_3}{H_1^2} \qquad k_{17} = \frac{-3\bar{G}_1H_3}{H_1}\frac{\partial}{\partial \bar{x}}$$

$$k_{22} = 4(\bar{E}_1 + \bar{E}_2)\frac{\partial^2}{\partial \bar{x}^2} - \left(\frac{12\bar{G}_1}{H_1^2} + \frac{3\bar{G}_2}{H_2^2}\right)$$

$$k_{23} = -(2\bar{E}_1 + \bar{E}_2)\frac{\partial^2}{\partial \bar{x}^2} + \left(\frac{6\bar{G}_1}{H_1^2} + \frac{3\bar{G}_2}{H_2^2}\right) \qquad k_{24} = 0$$

$$k_{25} = 0 \qquad k_{26} = (2\bar{E}_1 + \bar{E}_2)H_3\frac{\partial^2}{\partial \bar{x}^2} - \left(\frac{6\bar{G}_1}{H_1^2} + \frac{3\bar{G}_2}{H_2^2}\right)H_3$$

$$k_{27} = \left(\frac{6\bar{G}_1}{H_1} - \frac{3\bar{G}_2}{H_2}\right)\frac{\partial}{\partial \bar{x}}$$

$$k_{33} = (1 + 2\bar{E}_3)\frac{\partial^2}{\partial \bar{x}^2} - 3\left(\frac{\bar{G}_1}{H_1^2} + \frac{\bar{G}_2}{H_2^2} + \frac{\bar{G}_4}{H_4^2} + \frac{\bar{G}_5}{H_5^2}\right)$$

$$k_{34} = -(\bar{E}_4 + 2\bar{E}_5)\frac{\partial^2}{\partial \bar{x}^2} + \left(\frac{3\bar{G}_3}{H_4^2} + \frac{6\bar{G}_5}{H_5^2}\right)$$

$$k_{35} = \bar{E}_5\frac{\partial^2}{\partial \bar{x}^2} - \frac{3\bar{G}_5}{H_5^3}$$

$$k_{36} = -(\bar{E}_1 + \bar{E}_2 - \bar{E}_4 - \bar{E}_5)\frac{\partial^2}{\partial \bar{x}^2} + 3\left(\frac{\bar{G}_1}{H_1^2} + \frac{\bar{G}_2}{H_2^2} - \frac{\bar{G}_4}{H_4^2} - \frac{\bar{G}_5}{H_5^2}\right)H_3$$

$$k_{37} = -3\left(\frac{\bar{G}_1}{H_1} - \frac{\bar{G}_2}{H_2} + \frac{\bar{G}_4}{H_4} - \frac{\bar{G}_5}{H_5}\right)\frac{\partial}{\partial \bar{x}}$$

$$k_{44} = 4(\bar{E}_4 + \bar{E}_5)\frac{\partial^2}{\partial \bar{x}^2} - \left(\frac{3\bar{G}_4}{H_4^2} + \frac{12\bar{G}_5}{H_5^2}\right)$$

$$k_{45} = -2\bar{E}_5\frac{\partial^2}{\partial \bar{x}^2} + \frac{6\bar{G}_5}{H_5^2}$$

$$k_{46} = -(\bar{E}_4 + 2\bar{E}_5)H_3\frac{\partial^2}{\partial \bar{x}^2} + \left(\frac{3\bar{G}_4}{H_4^2} + \frac{6\bar{G}_5}{H_5^2}\right)H_3$$

$$k_{47} = \left(\frac{3\bar{G}_4}{H_4} - \frac{3\bar{G}}{H_5}\right)\frac{\partial}{\partial \bar{x}} \qquad k_{55} = 4\bar{E}_5\frac{\partial^2}{\partial \bar{x}^2} - \frac{3\bar{G}_5}{H_5^2}$$

$$k_{56} = \left(\bar{E}_5\frac{\partial^2}{\partial \bar{x}^2} - \frac{3\bar{G}_5}{H_5^2}\right)H_3 \qquad k_{57} = \frac{3\bar{G}_5}{H_5}\frac{\partial}{\partial \bar{x}}$$

$$k_{66} = H_3^2\frac{\partial^2}{\partial \bar{x}^2} - 3\left(\frac{\bar{G}_1}{H_1^2} + \frac{\bar{G}_2}{H_2^2} + \frac{\bar{G}_3}{H_3^2} + \frac{\bar{G}_4}{H_4^2} + \frac{\bar{G}_5}{H_5^2}\right)H_3^2$$

$$k_{67} = 3\left(\frac{\bar{G}_1}{H_1} - \frac{\bar{G}_2}{H_2} + \frac{\bar{G}_3}{H_3} - \frac{\bar{G}_4}{H_4} + \frac{\bar{G}_5}{H_5}\right)H_3\frac{\partial}{\partial \bar{x}} \qquad k_{77} = 3\bar{G}\frac{\partial^2}{\partial \bar{x}^2}$$

and the entries satisfy

$$k_{ij} = k_{ji}, \qquad i = 1,\dots,6, \quad j = 1,\dots,6$$

and

$$k_{i7} = -k_{7i}, \qquad i = 1,\dots,6$$

The dimensionless boundary conditions at $\bar{x} = 0$ are

$$\{Y\} = \{\bar{u}_1, \bar{u}_2, \ldots, \bar{u}_5, \psi_3, \bar{w}\}^T = \{0\}^T \tag{7}$$

The dimensionless boundary conditions for the sandwich part (section 1) at $\bar{x} = l$ are

$$N_1^{(1)} = 4\bar{E}_1\frac{\partial \bar{u}_1}{\partial \bar{x}} - 2\bar{E}_1\frac{\partial \bar{u}_2}{\partial \bar{x}} + \bar{E}_1\frac{\partial \bar{u}_3}{\partial \bar{x}} - \bar{E}_1 H_3\frac{\partial \psi_3}{\partial \bar{x}} = 0 \tag{8a}$$

$$N_2^{(1)} = \Big[-2\bar{E}_1\frac{\partial \bar{u}_1}{\partial \bar{x}} + 4(\bar{E}_1 + \bar{E}_2)\frac{\partial \bar{u}_2}{\partial \bar{x}} - (2\bar{E}_1 + \bar{E}_2)\frac{\partial \bar{u}_3}{\partial \bar{x}} + (2\bar{E}_1 + \bar{E}_2)H_3\frac{\partial \psi_3}{\partial \bar{x}}\Big] = 0 \tag{8b}$$

$$N_3^{(1)} = \bar{E}_1\frac{\partial \bar{u}_1}{\partial \bar{x}} - (2\bar{E}_1 + \bar{E}_2)\frac{\partial \bar{u}_2}{\partial \bar{x}} + (1 + 2\bar{E}_3)\frac{\partial \bar{u}_3}{\partial \bar{x}} - (\bar{E}_4 + 2\bar{E}_5)\frac{\partial \bar{u}_4}{\partial \bar{x}} + \bar{E}_5\frac{\partial \bar{u}_5}{\partial \bar{x}} - (\bar{E}_1 + \bar{E}_2 - \bar{E}_4 - \bar{E}_5)H_3\frac{\partial \psi_3}{\partial \bar{x}} \tag{8c}$$

$$N_4^{(1)} = \Big[-(\bar{E}_4 + 2\bar{E}_5)\frac{\partial \bar{u}_3}{\partial \bar{x}} + 4(\bar{E}_4 + \bar{E}_5)\frac{\partial \bar{u}_4}{\partial \bar{x}} - 2\bar{E}_5\frac{\partial \bar{u}_5}{\partial \bar{x}} - (\bar{E}_4 + 2\bar{E}_5)H_3\frac{\partial \psi_3}{\partial \bar{x}}\Big] = 0 \tag{8d}$$

$$N_5^{(1)} = \bar{E}_5\frac{\partial \bar{u}_3}{\partial \bar{x}} - 2\bar{E}_5\frac{\partial \bar{u}_4}{\partial \bar{x}} + 4\bar{E}_5\frac{\partial \bar{u}_5}{\partial \bar{x}} + \bar{E}_5 H_3\frac{\partial \psi_3}{\partial \bar{x}} = 0 \tag{8e}$$

$$M^{(1)} = -\bar{E}_1 H_3\frac{\partial \bar{u}_1}{\partial \bar{x}} + (2\bar{E}_1 + \bar{E}_2)H_3\frac{\partial \bar{u}_2}{\partial \bar{x}} - (\bar{E}_1 + \bar{E}_2 - \bar{E}_4 - \bar{E}_5)H_3\frac{\partial \bar{u}_3}{\partial \bar{x}} - (\bar{E}_4 + 2\bar{E}_5)H_5\frac{\partial \bar{u}_4}{\partial \bar{x}} + \bar{E}_5 H_3\frac{\partial \bar{u}_5}{\partial \bar{x}} + \bar{E}_5 H_3\frac{\partial \psi_3}{\partial \bar{x}} \tag{8f}$$

$$S^{(1)} = 3\bar{G}\frac{\partial \bar{w}}{\partial \bar{x}} + \frac{3\bar{G}_1}{H_1}\bar{u}_1 + \left(\frac{3\bar{G}_2}{H_2} - \frac{6\bar{G}_1}{H_1}\right)\bar{u}_2 + \left(\frac{3\bar{G}_1}{H_1} - \frac{3\bar{G}_2}{H_2} + \frac{3\bar{G}_4}{H_4} - \frac{3\bar{G}_5}{H_5}\right)\bar{u}_3 + \left(\frac{3\bar{G}_5}{H_5} - \frac{3\bar{G}_4}{H_4}\right)\bar{u}_4 - \frac{3\bar{G}_5}{H_5}\bar{u}_5 - 3\left(\frac{\bar{G}_1}{H_1} - \frac{\bar{G}_2}{H_2} + \frac{\bar{G}_3}{H_3} - \frac{\bar{G}_4}{H_4} + \frac{\bar{G}_5}{H_5}\right)H_3\psi_3 \tag{8g}$$

where $\bar{G} = \bar{G}_1 + \bar{G}_2 + \bar{G}_3 + \bar{G}_4 + \bar{G}_5$. Here the $N_i^{(1)}$, $M^{(1)}$, and $S^{(1)}$ represent the generalized axial force in each layer, the generalized moment for the cross section, and the generalized shear for the cross section, respectively, in the sandwich part of the beam.

The differential equations of motion for the uncovered portion of the beam (section 2) are

$$-M_3\frac{\partial^2 \bar{u}_3'}{\partial \bar{t}^2} + \bar{E}_3\frac{\partial^2 \bar{u}_3'}{\partial \bar{x}^2} = 0 \tag{9a}$$

$$-M_3 H_3^2\frac{\partial^2 \psi_3'}{\partial \bar{t}^2} + \bar{E}_3 H_3^2\frac{\partial^2 \psi_3'}{\partial \bar{x}^2} + 3\bar{G}_3\left(\frac{\partial \bar{w}'}{\partial \bar{x}} - \psi_3'\right) = 0 \tag{9b}$$

$$-M_3\frac{\partial^2 \bar{w}'}{\partial \bar{t}^2} + \bar{G}_3\left(\frac{\partial^2 \bar{w}'}{\partial \bar{x}^2} - \frac{\partial \psi_3'}{\partial \bar{x}}\right) = 0 \tag{9c}$$

All primed quantities $(\,)'$ represent the variables in the uncovered portion of the beam. The boundary conditions for the uncovered portion of the beam at $\bar{x} = l$ are

$$N_3^{(2)} = 3\bar{E}_3\frac{\partial \bar{u}_3'}{\partial \bar{x}} \tag{10a}$$

$$M^{(2)} = \bar{E}_3 H_3^2\frac{\partial \psi_3'}{\partial \bar{x}} \tag{10b}$$

$$S^{(2)} = -3\bar{G}_3\left(\frac{\partial \bar{w}'}{\partial \bar{x}} - \psi_3'\right) \tag{10c}$$

The interface between section 1 (covered part of the beam) and section 2 (uncovered part of the beam) requires that $S^{(1)} + S^{(2)} = 0$, $M^{(1)} + M^{(2)} = 0$, and $N_3^{(1)} + N_3^{(2)} = 0$ at $\bar{x} = l$. Also, the displacement and slope at the interface must be continuous:

$$\bar{w} = \bar{w}', \qquad \frac{\partial \bar{w}}{\partial \bar{x}} - \psi_3 = \frac{\partial \bar{w}'}{\partial \bar{x}} - \psi_3'$$

The dimensionless boundary conditions at $\bar{x} = 1$ are

$$\left\{\frac{\partial \bar{u}_3'}{\partial \bar{x}}, \frac{\partial \psi_3'}{\partial \bar{x}}, \frac{\partial \bar{w}'}{\partial \bar{x}} - \psi_3'\right\} = \{0, 0, 0\} \tag{11}$$

indicating that no extensional force, moment, and shear force exist at the end of the beam.

Frequency and Loss Factor

To solve the governing differential equation of motion of the system, we assume

$$Y = Y_n e^{\lambda^* \bar{x}} e^{j\Omega_n^* \bar{t}} \tag{12}$$

where Y and Y_n are defined as

$$Y = [\bar{u}_1, \bar{u}_2, \ldots, \bar{u}_5, \psi_3, \bar{w}]^T$$

$$Y_n = [\bar{u}_{1n}, \bar{u}_{2n}, \ldots, \bar{u}_{5n}, \psi_{3n}, \bar{w}_n]^T$$

and λ^* depends upon the value of Ω_n^*, as discussed below. Substituting Y into the governing Eq. (6), nontrivial solutions exist if the corresponding determinant is zero. Thus

$$\det\big(-[M]\Omega_n^{*2} + [K]\big) = 0 \tag{13}$$

where the $\partial/\partial\bar{x}$ and $\partial^2/\partial\bar{x}^2$ terms in $[K]$ are replaced by λ^* and λ^{*2}, respectively. Similarly, for section 2, we have the following characteristic equation generated from Eq. (9):

$$M_3\Omega_n^{*2} + \bar{E}_3\lambda^{*2} = 0 \tag{14a}$$

$$\det\begin{Bmatrix} M_3 H_3^2\Omega_n^{*2} + \bar{E}_3 H_3^2\lambda^{*2} + 3\bar{G}_3 & 3\bar{G}_3\lambda^* \\ -3\bar{G}_3\lambda^* & 3\Omega_n^{*2} + 3\bar{G}_3\lambda^{*2}\end{Bmatrix} = 0 \tag{14b}$$

where λ^* are characteristic values. For any Ω_n^*, let $\lambda_{n,1}^*, \lambda_{n,2}^*, \ldots, \lambda_{n,14}^*$ denote the 14 solutions of Eq. (13); let $\lambda_{n,15}^*, \lambda_{n,16}^*, \lambda_{n,17}^*, \lambda_{n,18}^*$ denote the four zeros of Eq. (14b); and let $\lambda_{n,19}^*$, and $\lambda_{n,20}^*$ denote the two zeros of Eq. (14a). These values may be determined from the boundary conditions and continuity conditions.

For section 2 (the uncovered part), we have

$$\bar{w}_n'(\bar{x}) = \sum_{i=15}^{18} A_{ni} e^{\lambda_{n,i}^* \bar{x}} \qquad \psi_{3n}'(\bar{x}) = \sum_{i=15}^{18} A_{n,i} e^{\lambda_{n,i}^* \bar{x}}$$

$$\bar{u}_{3n}'(\bar{x}) = \sum_{i=19}^{20} A_{ni} e^{\lambda_{n,i}^* \bar{x}} \tag{15}$$

For section 1 (the covered part),

$$\bar{u}_{1n}(\bar{x}) = \sum_{i=1}^{14} A_{ni} U_{1i} e^{\lambda_{n,i}^* \bar{x}} \qquad u_{2n}(\bar{x}) = \sum_{i=1}^{14} A_{ni} U_{2i} e^{\lambda_{n,i}^* \bar{x}}$$

$$u_{3n}(\bar{x}) = \sum_{i=1}^{14} A_{ni} U_{3i} e^{\lambda_{n,i}^* \bar{x}} \qquad u_{4n}(\bar{x}) = \sum_{i=1}^{14} A_{ni} U_{4i} e^{\lambda_{n,i}^* \bar{x}} \tag{16}$$

$$u_{5n}(\bar{x}) = \sum_{i=1}^{14} A_{ni} U_{5i} e^{\lambda_{n,i}^* \bar{x}} \qquad \bar{w}_n(\bar{x}) = \sum_{i=1}^{14} A_{ni} e^{\lambda_{n,i}^* \bar{x}}$$

$$\psi_{3n}(\bar{x}) = \sum_{i=1}^{14} A_{ni}\phi_i e^{\lambda_{n,i}^* \bar{x}}$$

The $A_i s$ ($i = 1, 2, \ldots, 20$) have to be determined from the boundary conditions and the continuity conditions at the interface between the covered part and the uncovered part. We have a total of 20 homogeneous equations involving the unknowns A_i. For nontrivial solution, the corresponding determinant must be zero, i.e.,

$$\det(\Omega_n^*, \lambda^*) = 0 \tag{17}$$

Equations (13), (14), and (17) are nonlinear complex equations for unknowns Ω_n^* and λ^* (the characteristic value). We define the complex resonance frequency factor p^*, the real resonance frequency factor p, and the system loss factor η of the beam as follows:

$$p^* = \Omega_n^* t_0 = p\sqrt{1 + j\eta} \qquad p = \Omega_n t_0 = \sqrt{\mathrm{Re}(p^{*2})}$$
$$\eta = \frac{\mathrm{Im}(p^{*2})}{\mathrm{Re}(p^{*2})} \tag{18}$$

where $j = (-1)^{1/2}$.

Comparisons of Numerical Results

Numerical results for different parameters were obtained and are displayed as graphs. The input parameters employed in the previously described numerical scheme, unless stated otherwise, were

$$l = L_1/L = 0.2, \qquad \rho_1 = \rho_3 = \rho_5 = 7850.0 \text{ kg/m}^3$$
$$\rho_2 = \rho_4 = 3140.0 \text{ kg/m}^3, \qquad H_3 = 0.12$$
$$H_1 = H_2 = H_4 = H_5 = 0.02, \qquad \bar{E}_1 = \bar{E}_3 = \bar{E}_5 = 0.30$$
$$\bar{E}_2 = \bar{E}_4 = 3\bar{G}_2, \qquad \bar{G}_3 = \bar{G}_5 = \bar{E}_1/2.5$$
$$\bar{G}_2 = \bar{G}_4 = G_{r2}(1 + j\eta_2), \qquad G_{r2} = 0.001$$
$$\eta_2 = \eta_4 = 1.2, \qquad k_i' = 0.833$$

The values of η_2 and η_4 are representative of high damping materials and G_{r2} is the real part of $\bar{G}_2$. The values of k_1'–k_5' are taken as being equal to 0.833 for the sake of simplicity. Finally, the $E_i s$ and $G_i s$ are connected via the Poisson ratio. The Poisson ratio of the primary beam is assumed to be 0.25 and the Poisson ratio of the damping layers is assumed to be 0.5.

Effect of Damping Layer Height

Figure 3 shows the system loss factor vs the damping layer height ratio (the height of the damping layer to the length of the beam, H_2 or H_4). In general, the system loss factor for the improved model is less than that for the Euler beam model.[14]

Figure 4 shows the resonance frequency factor vs the damping layer height ratio. It can be seen that, for small ratios of H_2, the frequency factor for the two models is almost the same. The difference between the two models increases with an increase in the ratio H_2. We note that the resonance frequency factor predicted by the improved model is smaller than that predicted by Euler beam model. This trend coincides with results reported by Weaver et al.,[18] where no damping treatment was present.

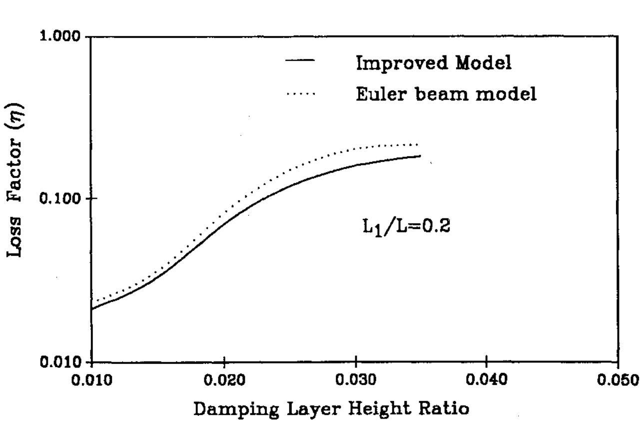

Fig. 3 Loss factor vs damping layer height ratio.

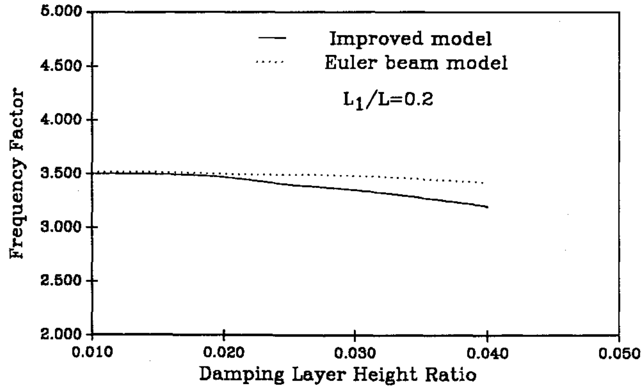

Fig. 4 Frequency factor vs damping layer height ratio.

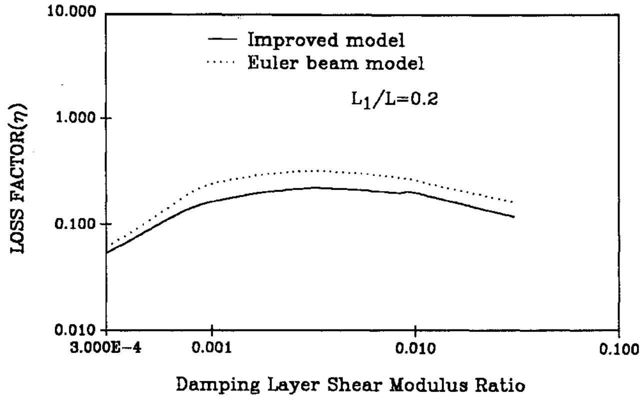

Fig. 5 Loss factor vs damping layer shear modulus ratio.

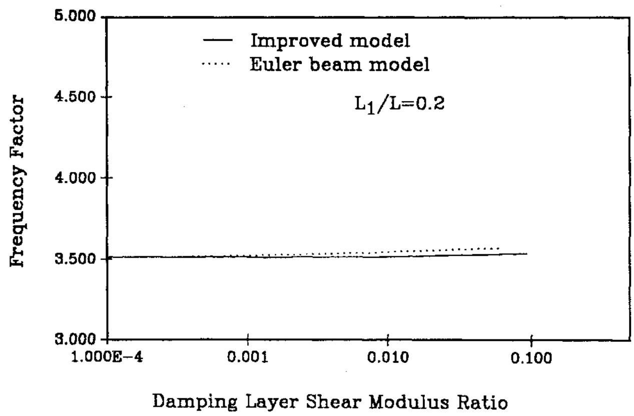

Fig. 6 Frequency factor vs damping layer shear modulus ratio.

Effect of Damping Layer Shear Modulus

Figure 5 shows the results of system loss factor vs damping layer shear modulus ratio. There exists a damping layer shear modulus ratio that will produce an optimal system loss factor. This coincides with previous results.[2,14] We note that the Euler beam model gives a larger value of system loss factor compared with the improved model. In other words, the Euler beam model overestimates the system loss factor.

Figure 6 shows the results of resonance frequency factor vs damping layer shear modulus ratio. There are two different trends to be noted: The first is between models, and the second is a general trend. The resonance frequency factor predicted by the improved model is smaller than that predicted by the Euler beam model. We note that for small shear modulus ratio, the two models give almost the same results. However, for large shear modulus ratio, the improved model gives lower resonance frequency factors. The difference between the models increases with the increase in shear modulus ratio. For the improved model, the effects of rotatory and extensional inertia as well as bending, extension, and shear deformation in the damping layers are included, whereas for the Euler beam model, only the

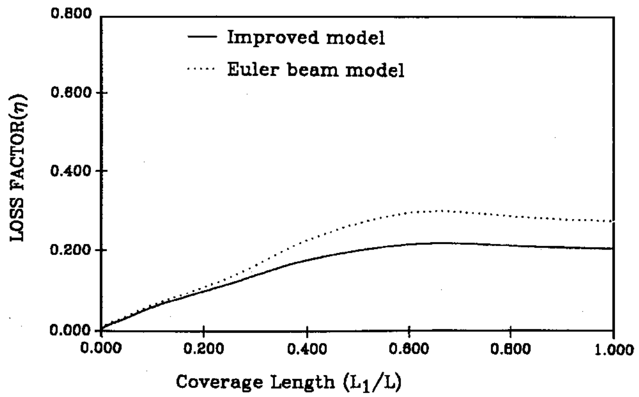

Fig. 7 Loss factor vs coverage length ratio.

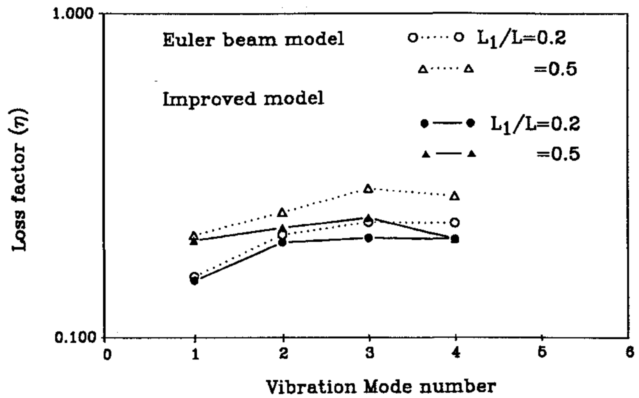

Fig. 8 Loss factor vs vibration mode for various coverages.

shear effects in the damping layers are accounted for. If the value of damping layer shear modulus ratio is small, the shear deformation effects are small and may be neglected. However, for large values of damping layer shear modulus, we need to take these effects into account. The general trend derived from Fig. 6 is that an increase in shear modulus of the damping layers will increase the bending stiffness of the entire system and causes an increase in the frequency factor for both models.

Effect of Sandwich Coverage Length

Figure 7 shows the system loss factor vs coverage length ratio of the sandwich part ($l = L_1/L$). In the case of no damping coverage ($l = 0$), both models give zero loss factor. For small value of coverage ratio, the two models give almost the same results. As the coverage increases, differences in the system loss factor between the two models occur with Euler beam model predicting higher values of system loss factor. We note that an increase in coverage will increase the system loss factor to some maximum value. This trend follows the trend of the covered Euler beam model.[14]

Effect of Vibration Mode Number

Figure 8 shows the results of system loss factor vs the mode number. In general, the Euler beam model predicts higher values of system loss factor compared with the improved model. We note that even the first mode predicts differences between the two models, but the difference is small compared with the higher modes. Coverage length also affects the difference in the system loss factor, and the effects are more pronounced in the first few modes.

Conclusions

For the double-sandwich cantilever beam, numerical results show that there will be differences between the results of the improved beam model and the Euler beam model in the following situations: 1) stiff damping layers; 2) large partially or fully covered sandwich beam; 3) large thickness of damping layers compared with the thickness of primary beam; and 4) higher modes of vibration. In these cases, it may be better to use the results from the improved beam model. The differences exist as a result of the effects of bending and extension, rotational and longitudinal inertia of the damping layers, and the shear deformation and rotational and longitudinal inertia of the constraining layers and primary beam, which are included in the governing differential equations. The theory given here is applicable for long or short, soft or stiff damping layers double sandwich beams.

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